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## AVERAGE AND PROBABILITY.

195. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue Philadelphia, Pa.

A random diameter is drawn in a given circle. Find the chance that it intersects, (1) a random chord; (2) a random chord through a random point; and (3) a chord through two random points.

Solution by the PROPOSER.

Let  $O$  be the center of the given circle,  $AB$  the random chord,  $M$  the one random point, and  $N$  the second random point.

Let  $OA=r$ ,  $AM=x$ ,  $\angle AOH=\theta$ . For the second point, let  $MN=y$  and let  $\mu=\text{angle } AB \text{ makes with some fixed line}$

Then  $AH=r\sin\theta$ ; an element of the circle at  $M$  is  $r\sin\theta d\theta dx$ ; at  $n$ ,  $d\mu ydy$ . The limits of  $\theta$  are 0 and  $\frac{1}{2}\pi$ ; of  $x$ , 0 and  $2r\sin\theta=x'$ ; of  $y$ , 0 and  $x$  and doubled. Then the required chance is

$$(1) \quad p = \frac{\int_0^{\frac{1}{2}\pi} 4\theta d\theta}{\int_0^{\frac{1}{2}\pi} 2\pi d\theta} = \frac{4}{\pi^2} \int_0^{\frac{1}{2}\pi} \theta d\theta = \frac{1}{2}.$$

$$(2) \quad p = \frac{\int_0^{\frac{1}{2}\pi} \int_0^{x'} 4r\theta \sin\theta d\theta dx}{\int_0^{\frac{1}{2}\pi} \int_0^{x'} 2\pi r\sin\theta d\theta dx} = \frac{4}{\pi^2 r} \int_0^{\frac{1}{2}\pi} \int_0^{x'} \theta \sin\theta d\theta dx$$

$$= \frac{8}{\pi^2} \int_0^{\frac{1}{2}\pi} \theta \sin^2\theta d\theta = \frac{1}{2} + \frac{2}{\pi^2}.$$

$$(3) \quad p = \frac{\int_0^{\frac{1}{2}\pi} \int_0^{x'} \int_0^x \int_0^{2\pi} 4r\theta \sin\theta d\theta dx d\mu ydy}{\int_0^{\frac{1}{2}\pi} \int_0^{x'} \int_0^x \int_0^{2\pi} 2\pi r\sin\theta d\theta dx d\mu ydy}$$

$$= \frac{4}{\pi^3 r^3} \int_0^{\frac{1}{2}\pi} \int_0^{x'} \int_0^x \int_0^{2\pi} \theta y \sin\theta d\theta dx dy d\mu$$

$$= \frac{4}{\pi^2 r^3} \int_0^{\frac{1}{2}\pi} \int_0^{x'} x^2 \theta \sin\theta d\theta dx = \frac{32}{3\pi^2} \int_0^{\frac{1}{2}\pi} \theta \sin^4\theta d\theta = \frac{1}{2} + \frac{8}{3\pi^2}.$$

If a random chord is drawn instead of a random diameter, we let  $\angle COK=\phi$ ,  $\angle AOC=\psi$ . The limits of  $\phi$  are 0 and  $\theta$  and doubled; of  $\psi$ , 0 and  $2\phi$  and doubled.

$$\begin{aligned} \therefore (1) \quad p &= \frac{\int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} d\theta d\phi d\psi}{\int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^\pi d\theta d\phi d\psi} = \frac{8}{\pi^3} \int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} d\theta d\phi d\psi \\ &= \frac{16}{\pi^3} \int_0^{\frac{1}{2}\pi} \int_0^\theta \phi d\theta d\phi = \frac{8}{\pi^3} \int_0^{\frac{1}{2}\pi} \theta^2 d\theta = \frac{1}{3} \text{ (same as 191).} \end{aligned}$$

$$\begin{aligned} (2) \quad p &= \frac{\int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} \int_0^{x'} r \sin \theta d\theta d\phi d\psi dx}{\int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^\pi \int_0^{x'} r \sin \theta d\theta d\phi d\psi dx} \\ &= \frac{8}{\pi r(\pi^2 + 4)} \int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} \int_0^{x'} \sin \theta d\theta d\phi d\psi dx \\ &= \frac{16}{\pi(\pi^2 + 4)} \int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} \sin^3 \theta d\theta d\phi d\psi = \frac{16}{\pi(\pi^2 + 4)} \int_0^{\frac{1}{2}\pi} \theta^2 \sin^3 \theta d\theta \\ &= \frac{\pi^2 + 6}{3(\pi^2 + 4)} = \frac{1}{3} + \frac{2}{3(\pi^2 + 4)}. \end{aligned}$$

$$\begin{aligned} (3) \quad p &= \frac{\int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} \int_0^{x'} \int_0^x y r \sin \theta d\theta d\phi d\psi dx dy}{\int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^\pi \int_0^{x'} \int_0^x y r \sin \theta d\theta d\phi d\psi dx dy} \\ &= \frac{48}{\pi r^3 (3\pi^2 + 16)} \int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} \int_0^{x'} \int_0^x y \sin \theta d\theta d\phi d\psi dx dy \\ &= \frac{64}{\pi (3\pi^2 + 16)} \int_0^{\frac{1}{2}\pi} \int_0^\theta \int_0^{2\phi} \sin^4 \theta d\theta d\phi d\psi = \frac{64}{\pi (3\pi^2 + 16)} \int_0^{\frac{1}{2}\pi} \theta^2 \sin^4 \theta d\theta \\ &= \frac{\pi^2 + 9}{3\pi^2 + 16} = \frac{1}{3} + \frac{11}{3(3\pi^2 + 16)}. \end{aligned}$$

This exhibits the reason for the  $\frac{1}{3}$  given by some contributors to 191 instead of the correct value  $\frac{1}{3}$ .